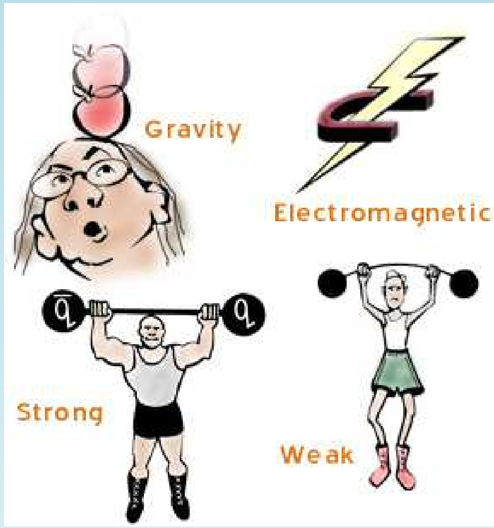
Lattice QCD:

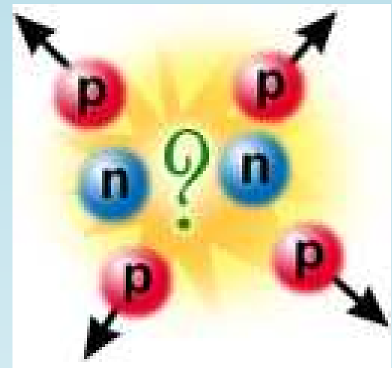
Simulating Quarks on a Computer

Tereza Mendes

Instituto de Física de São Carlos – USP

Quantum Chromodynamics (QCD)

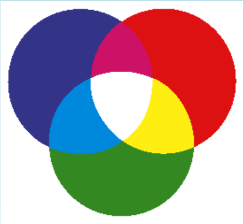




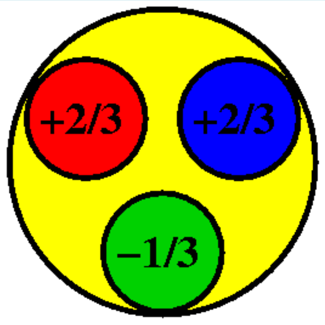
- The Standard Model describes 3 (of the 4 known) fundamental interactions of nature: the electromagnetic, the weak and the strong forces. It has been fully confirmed experimentally, except for the detection of the Higgs boson.
- The strong interaction between quarks is described by QCD, a quantum field theory based on the color charge, having three possible values and carried by gluons.

The Mystery of the Proton

The strong interaction between protons and neutrons is the residue of the interaction between the quarks that form them. The nucleons are formed by three quarks of different color.



The proton is a bound state of quarks interacting through the exchange of (massless) gluons.



Peculiar properties:

- Unusual bound state: 99% of the mass comes from the interaction! $\Rightarrow$ we are not star dust, we're gluons!
- Quarks are confined

The Practical Problem

QCD must be studied in a nonperturbative way

$\Rightarrow$ Lattice QCD, a strategy introduced by K. Wilson in 1974, based on similarity with statistical mechanics

Recent high-precision calculation:
Ab Initio Determination of Light Hadron Masses,
S.Dürr et al., Science 322, 1224 (2008).

Repercussion (November 2008):

- United Press
- Scientific American
- Nature (F. Wilczek)

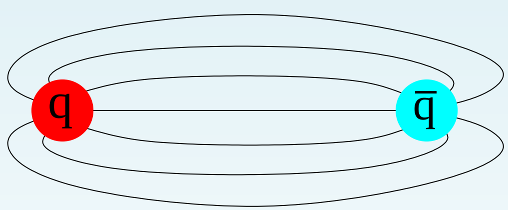
QCD vs. QED

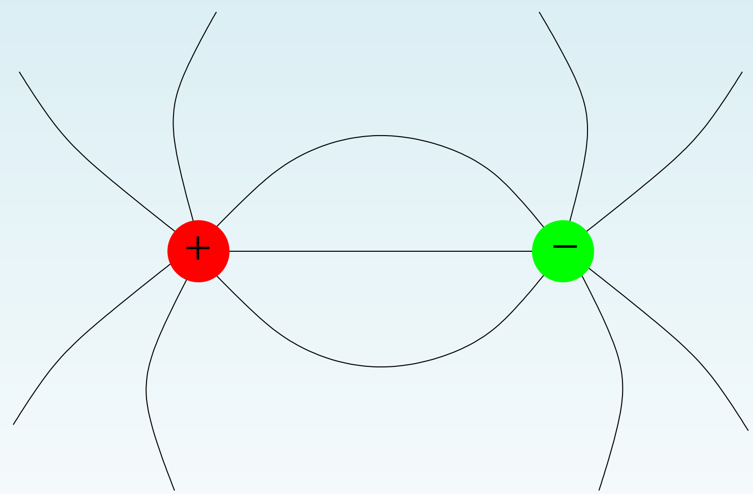
QCD (strong force) vs. QED (EM force)

quarks, gluons | eletrons, photons

$SU(3)$ (3 "colors") | $U(1)$

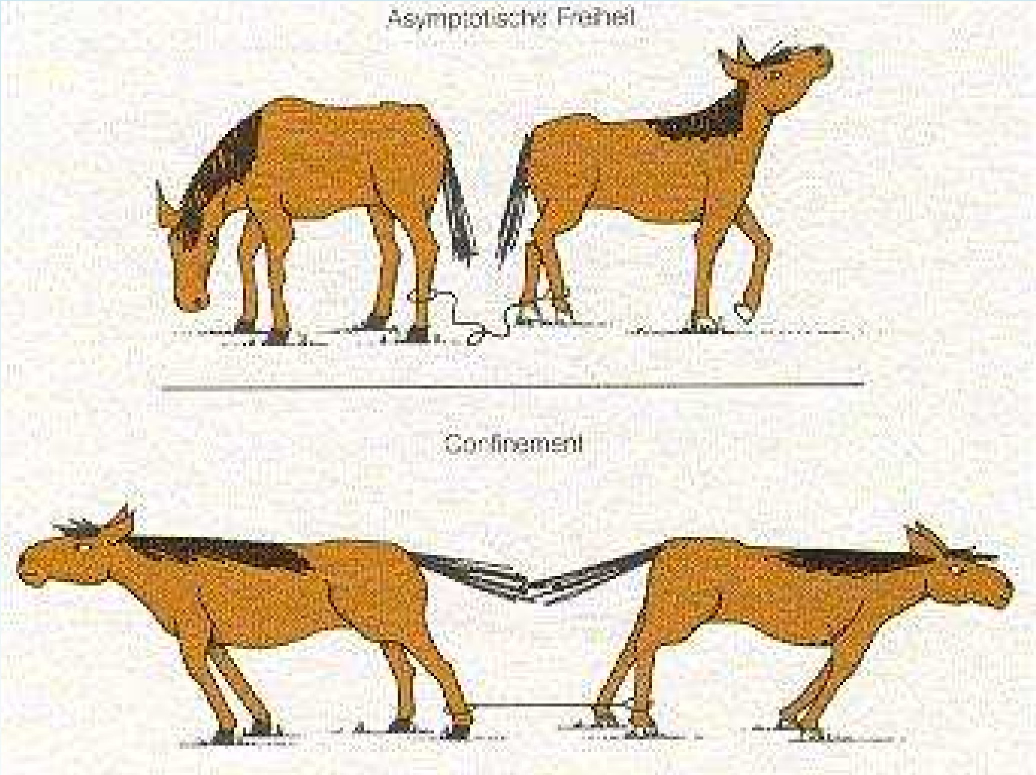
m_q, $\alpha_s(p)$ | m_e, $\alpha \approx 1/137$





Confinement vs. asymptotic freedom

At high energies (small distances), quarks are free, but at large distances the force becomes constant and we would need infinite energy to separate two quarks.



Lattice QCD

Lattice used by Kenneth G. Wilson in 1974 [*Confinement of quarks*, Phys. Rev. D 10, 2445 (1974)] as a trick to prove confinement in (strong-coupling) QCD. As he recalls

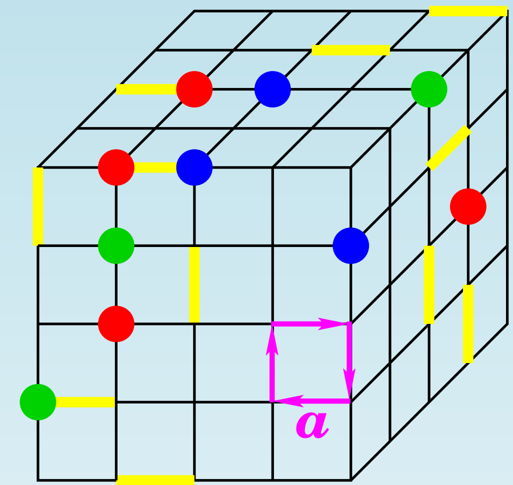
> *[...] Unfortunately, I found myself lacking the detailed knowledge and skills required to conduct research using renormalized non-Abelian gauge theories. What was I to do, especially as I was eager to jump into this research with as little delay as possible? [...] from my previous work in statistical mechanics I knew a lot about working with lattice theories...*
> *[...] I decided I might find it easier to work with a lattice version of QCD. . .*

The Origins of Lattice Gauge Theory, hep-lat/0412043 (Lattice 2004)

Lattice QCD Ingredients

Three ingredients

1. Quantization by path integrals $\Rightarrow$ sum over configurations with "weights" $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a \Rightarrow$ regularization



Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

The Wilson action

- is written in terms of the gauge links $U_{x,\mu} \equiv e^{i g_0 a A^b_\mu(x) T_b}$
- reduces to the usual action for $a \to 0$
- is gauge-invariant

The Lattice Action

The Wilson action (1974)

$$S = -\frac{\beta}{3} \sum_{\Box} \text{ReTr}\, U_{\Box}\,, \quad U_{x,\mu} \equiv e^{i g_0 a A_\mu^b(x) T_b}\,, \quad \beta = 6/{g_0}^2$$

- written in terms of oriented plaquettes formed by the link variables $U_{x,\mu}$, which are group elements
- under gauge transformations: $U_{x,\mu} \to g(x)\, U_{x,\mu}\, g^\dagger(x+\mu)$, where $g \in SU(3)$ $\Rightarrow$ closed loops are gauge-invariant quantities
- integration volume is finite: no need for gauge-fixing

At small β (i.e. strong coupling) we can perform an expansion analogous to the high-temperature expansion in statistical mechanics. At lowest order, the only surviving terms are represented by diagrams with “double” or “partner” links, i.e. the same link should appear in both orientations, since $\int dU\, U_{x,\mu} = 0$

Confinement and Area Law

Considering a rectangular loop with sides R and T (the Wilson loop) as our observable, the leading contribution to the observable's expectation value is obtained by "tiling" its inside with plaquettes, yielding the area law

$$< W(R,T) > \sim \ \beta^{RT}$$

But this observable is related to the interquark potential for a static quark-antiquark pair

$$< W(R,T) > = \ e^{-V(R)T}$$

We thus have $V(R) \sim \sigma R$, demonstrating confinement at strong coupling (small β)!

Problem: the physical limit is at large β...

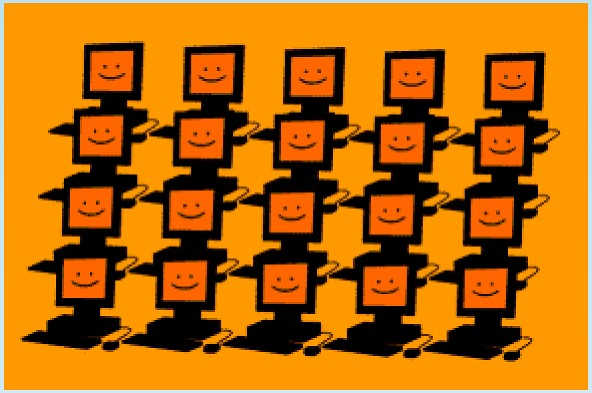
Numerical Lattice QCD

The approach had a "marvelous side effect", as Michael Creutz calls it

> *By discreetly making the system discrete, it becomes sufficiently well defined to be placed on a computer. This was fairly straightforward, and came at the same time that computers were growing rapidly in power. Indeed, numerical simulations and computer capabilities have continued to grow together, making these efforts the mainstay of lattice gauge theory.*

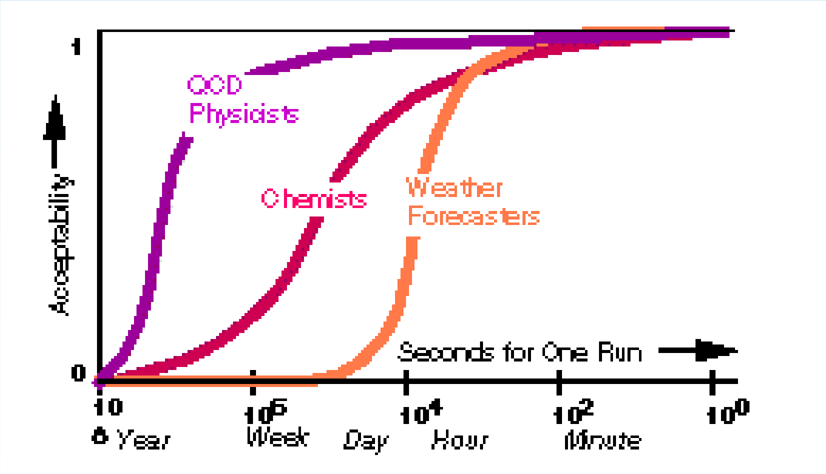
The Early days of lattice gauge theory,
AIP Conf. Proc. 690, 52 (2003)

Lattice QCD simulations



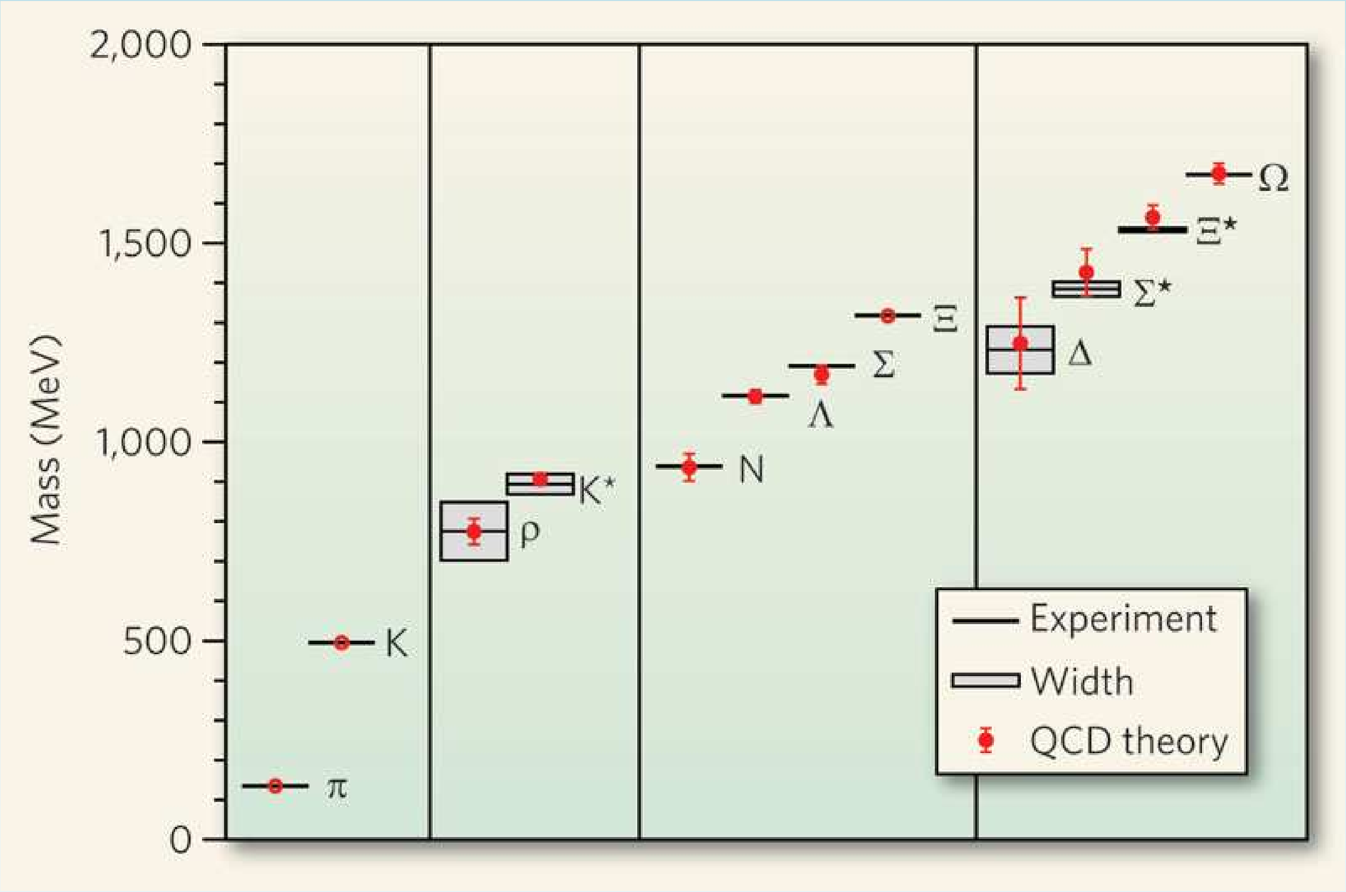
Due to the great complexity of the interaction and to the large number of degrees of freedom, the simulations described require considerable computational resources.

Simulations are usually done on powerful parallel supercomputers, some of which were designed and built specifically for the study of lattice QCD.



"QCD physicists have an extraordinary tolerance for execution times that take a significant fraction of a human lifetime"

Light-Hadron Spectrum

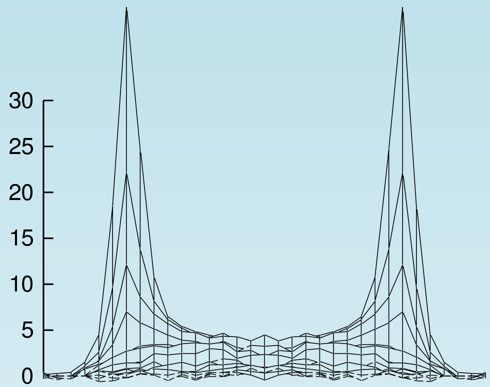


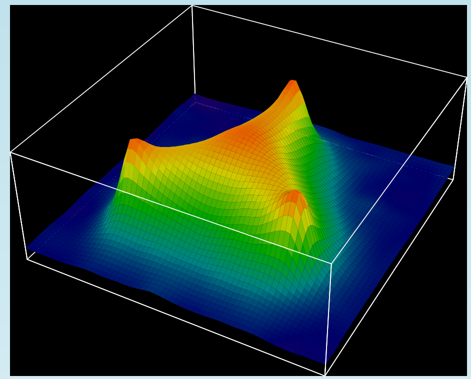
Masses of light hadrons computed by S. Dürr et al. (Science, 2008) versus experimental values. Note: π, K and Ξ used as inputs.

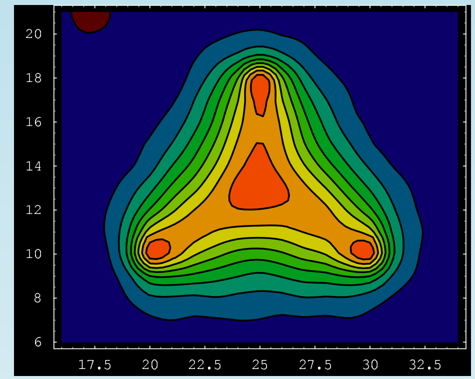
Cited by F. Wilczek in Nature 456, 449 (27 November 2008).

Confinement from Simulations

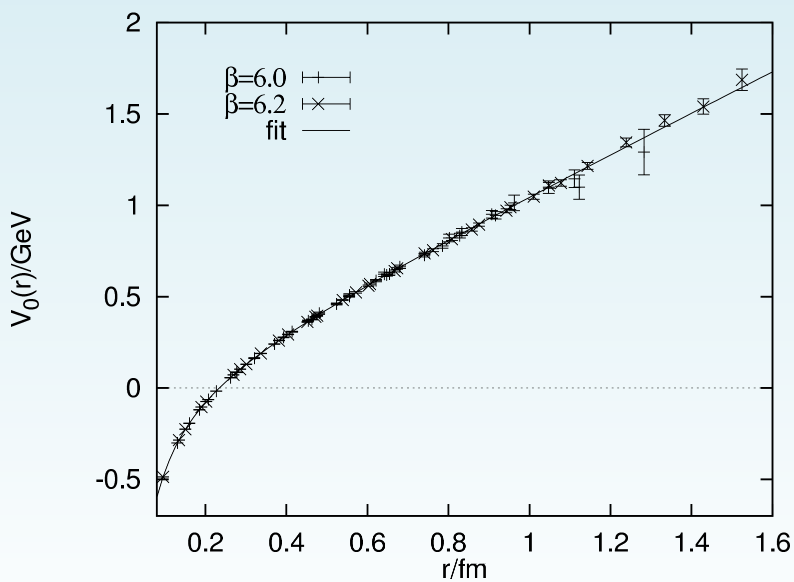
Can observe formation of flux tubes

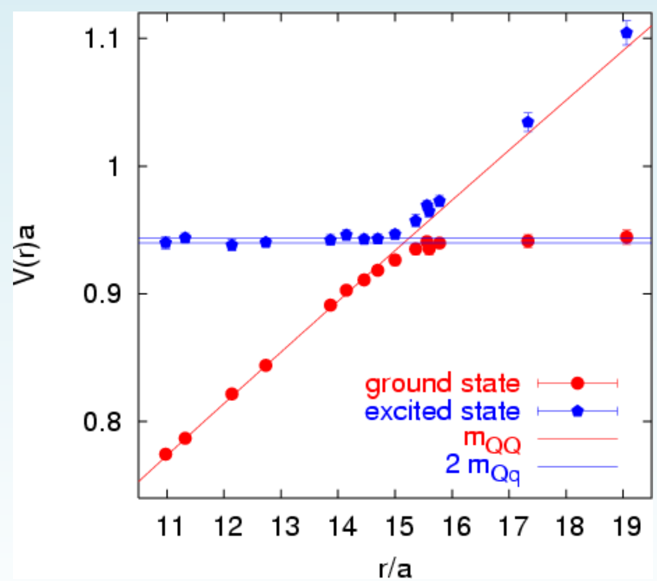






Linear rise of interquark potential, string breaking at large separation





General Thoughts

Now that we can calculate the proton's mass and we can visualize confinement in lattice simulations, are we done? No!

- Do we understand confinement? i.e., we know what it looks like, but do we know what it really is?
- Qualitative study of confinement ongoing at the IFSC–USP
- What about other predictions involving the strong force?

Heavy Quarks on the Lattice

- B physics is key to determining the flavor structure of the Standard Model through knowledge of the CKM matrix, describing quark mixing and CP violation, which may be associated with the lack of symmetry between matter and anti-matter in the universe.
- Theoretical inputs from numerical simulations of lattice QCD used in combination with experimental results.
- High precision is crucial! (but heavy-light systems are hard to represent on the lattice...)
- Fully nonperturbative treatment possible using HQET; first results including $1/m$ corrections

Conclusions

- 35 years after its proposal by K. Wilson, lattice QCD is now producing very precise results, which will be crucial to test the limits of the Standard Model, i.e. the limits of our ability to describe the visible universe.
- This study is heavily dependent on the interfaces between high-energy physics and two other scientific areas: statistical mechanics and computer science.
- The field of lattice QCD is not yet well represented in the southern hemisphere but is very advanced in Germany. I therefore benefitted very much from my stay at DESY–Zeuthen, and a short visit to CERN.

THANK YOU!